# **Topological Materials II**



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2016/May./26

# Outline

### 1. Introduction

Band theory Topology in condensed matter physics Basics properties: Robust, invariant number, gapless surface states Comparing with Landau's approach Density functional theory (DFT)

### 2. Topological insulator (quantum spin Hall insulator)

Strong topological insulator, weak topological insulator, topological crystalline insulator, topological Kondo insulator, quantum anomalous Hall effect...etc

### 3. Topological semimetal

3D Dirac semimetal, Weyl semimetal, Nodal-line semimetal, topological superconductor, New Fermion

## **Topological phases**



Blue: surface bands

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## **Weyl Fermion**



Equal numbers of  $\chi = +1$  and -1 WFs.

## Weyl semimetal

#### Weyl semimetals:

- 1. Provide the realization of Weyl fermions
- 2. Extend the classification of topological phases of matter beyond insulators
- 3. Magnetic monopole in k-space (topological number called "chiral charge")
- 4. Host exotic Fermi arc surface states



(1) robust (2) topological invariant number (3) gapless surface states

### Weyl semimetal (robust)

⇒k

**2D** 
$$H = k_1 \sigma_x + k_2 \sigma_y$$







 $H = k_1 \sigma_x + k_2 \sigma_y + m \sigma_z$ 

### Weyl semimetal (robust)

**2D** 
$$H = k_1 \sigma_x + k_2 \sigma_y$$

$$H = k_1 \sigma_x + k_2 \sigma_y + m \sigma_z$$

. 1

. 1



**3D** 
$$H = k_1\sigma_1 + k_2\sigma_2 + k_3\sigma_z$$
  
**Weyl**  

$$H = k_1\sigma_x + k_2\sigma_y + k_3\sigma_z + m\sigma_z$$

$$= k_1\sigma_x + k_2\sigma_y + \frac{(k_3 + m)\sigma_z}{k_3\sigma_z}$$

$$= k_1\sigma_x + k_2\sigma_y + \frac{k_3\sigma_z}{k_3\sigma_z}$$

TT

1

In the presence of translation invariant, small perturbation cannot destroy the node. But only move the node.

## Weyl semimetal (annihilate)



### Weyl semimetal (topological invariant number)

### *Chiral quantum number: χ*

Berry connection

$$\boldsymbol{A}(\boldsymbol{k}) = i \langle u(\boldsymbol{k}) | \boldsymbol{\nabla}_{\boldsymbol{k}} u(\boldsymbol{k}) \rangle$$

Berry curvature

$$\boldsymbol{F}(\boldsymbol{k}) = \boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{A}(\boldsymbol{k})$$

**Chiral Charge** 

$$\frac{1}{2\pi} \oint_{FS} \boldsymbol{F}(\boldsymbol{k}) \cdot \mathrm{d}\boldsymbol{S}(\boldsymbol{k}) = \chi$$

 $\chi = integer$ topological non-trivial

P. Hosur et al., C. R. Physique 14 (2013) 857

### Weyl semimetal (topological invariant number)

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P. Hosur et al., C. R. Physique 14 (2013) 857

Classical

Vector potential

### Α

Magnetic field

 $\mathbf{B}=\nabla\times\mathbf{A}$ 

Gauss's law for magnetism  $\oint \mathbf{B} \cdot d\mathbf{S} = 0$ 

### Weyl semimetal (pseduo-magnetic monopole)

Berry connection

$$\boldsymbol{A}(\boldsymbol{k}) = i \langle u(\boldsymbol{k}) | \boldsymbol{\nabla}_{\boldsymbol{k}} u(\boldsymbol{k}) \rangle$$

Berry curvature

$$\boldsymbol{F}(\boldsymbol{k}) = \boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{A}(\boldsymbol{k})$$

**Chiral Charge** 

$$\frac{1}{2\pi} \oint_{FS} \boldsymbol{F}(\boldsymbol{k}) \cdot \mathrm{d}\boldsymbol{S}(\boldsymbol{k}) = \chi$$

 $\chi = integer$ topological non-trivial

P. Hosur et al., C. R. Physique 14 (2013) 857



### Weyl semimetal (Fermi arc)





A:  $\chi=0 \Rightarrow$  topological trivial  $\Rightarrow$  no edge state B:  $\chi\neq0 \Rightarrow$  topological non-trivial  $\Rightarrow$  edge state

## Weyl semimetal (Fermi arc)



### Weyl semimetal (Fermi arc)



### Some features of Weyl semimetal

- 1. Gapless linear band
- 2. Spin singly degenerate
- 3. FS = discrete points
- 4. Non-zero chiral charge
- 5. Fermi arc surface state



A. A. Burkov et al., PRL 107, 127205 (2011)



 $m_{c1}^2 = (\Delta_S - \Delta_D)^2 < m^2 < m_{c2}^2 = (\Delta_S + \Delta_D)^2$ . Weyl semimetal  $m^2 < m_{c1}^2$  Band insulator  $m^2 > m_{c2}^2$  Topological insulator





J. Liu et al., PRB **90**, 155316 (2014)

 $LaBi_{1-x}Sb_xTe_3$  and  $LuBi_{1-x}Sb_xTe_3$  for  $x \approx 38.5 - 41.9\%$  and  $x \approx 40.5 - 45.1\%$  respectively.



## The light of hope

### ARTICLE

Received 24 Nov 2014 | Accepted 30 Apr 2015 | Published 12 Jun 2015

DOI: 10.1038/ncomms8373

OPEN

### A Weyl Fermion semimetal with surface Fermi arcs in the transition metal monopnictide TaAs class

PHYSICAL REVIEW X 5, 011029 (2015)

Weyl Semimetal Phase in Noncentrosymmetric Transition-Metal Monophosphides (Received 12 January 2015; published 17 March 2015)

### Weyl semimetal: TaAs



 $k_v(2\pi/a)$ 

### Weyl semimetal: TaAs



# Weyl semimetal: TaAs (ARPES)



### Weyl semimetal: NbP

EXP (STM)



### **Highlights of the Year**

#### Highlights of the Year

December 18, 2015 • Physics 8, 126

Physics picks its favorite stories from 2015.



#### **Discovery of Weyl Semimetals**

APS Physics ② physics 2015年7月24日 · @

**血說這專頁讚** 

Two research teams have confirmed the existence of Weyl fermionsmassless particles originally theorized as a solution to the Dirac equation. The findings are published in Science (http://go.aps.org/1gSk9EQ & http://go.aps.org/1gSI7RF) and in a forthcoming issue of APS's open access journal Physical Review X (http://go.aps.org/1JCGopq).

翻譯年糕



Weyl fermions are spotted at long last Solution to the Dirac equation detected 85 years after it was predicted PHYSICSWORLD.COM

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#### News

Discovery of Weyl fermion named a 'breakthrough of the year' by **Physics World magazine** 



Research A to Z | Contac

### **ScienceNews**

#### Elusive particle shows up in 'semimetal'

BY ANDREW GRANT 2:00PM JULY 16:2015 Magazine issue: Vol. 188, No. 4, August 22, 2015, p. 11

### chemistryworld

#### Elusive fermion found at long last

16 July 2015

Ida Emilie Steinmark

### TODAY AT BERKELEY LAB

the Top Ten Breakthroughs of the Year by

ability to behave as monopole and anti-

Zhong Fang and Hongming Weng of the

Chinese Academy of Sciences, for their pioneering work on Weyl fermions.

monopole inside a crystal

#### Physics World Names Weyl Fermion Research as a 'Top Ten Breakthrough of 2015'

DECEMBER 16 2015



Advanced Light Source user M. Zahid Hasan is one of three physicists whose efforts to observe Weyl fermions, an elusive massless particle theorized 85 years ago, were recognized by Physics World. Weyl fermions have been regarded as possible building blocks of other subatomic particles. The top 10 were chosen by a panel of Physics World editors and reporters. More>



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#### Weyl semimetal

From Wikipedia, the free encyclopedia

Weyl fermions are massless chiral fermions that play an important role in quantum field theory and the standard model. They may be thought of as a building block for fermions in quantum field theory, and were predicted from a solution to the Dirac equation derived by Hermann Weyl.<sup>[1]</sup> For example, one-half of a charged Dirac fermion of a definite chirality is a Weyl fermion.<sup>[2]</sup> They have not been observed as a fundamental particle in nature. Weyl fermions may be realized as emergent quasiparticles in a low-energy condensed matter system.<sup>[3][4]</sup>

Contents [hide] 1 Experimental discovery 2 Applications 3 Further reading 4 References

#### Experimental discovery [edit]

A **Weyl semimetal** is a solid state crystal whose low energy excitations are Weyl fermions.<sup>[6][7]</sup> A Weyl semimetal enables the first-ever realization of Weyl fermions.<sup>[8]</sup> It is a topologically nontrivial phase of matter that broadens the topological classification beyond topological insulators.<sup>[4]</sup> The Weyl fermions at zero energy correspond to points of bulk band degeneracy, the Weyl nodes that are separated in momentum space. Weyl fermions have distinct chiralities, either left handed or right handed. In a Weyl semimetal crystal, the chiralities associated with the Weyl nodes can be understood as topological invariant of this phase.<sup>[6]</sup> Comparing to the Dirac fermions in graphene or on the surface of topological insulators, Weyl fermions in a Weyl semimetal are the most robust electrons and do not depend on symmetries except the translation symmetry of the crystal lattice. Hence the Weyl fermion quasiparticles in a Weyl semimetal possess a



A schematic of the Weyl semimetal 53 state, which include the Weyl nodes and the Fermi arcs. The Weyl nodes are momentum space monopoles and anti-monopoles. The sketch is adapted from Ref.<sup>[5]</sup>

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PRB 58, 2788 (1998)

Quantum magnetoresistance

#### A. A. Abrikosov

Materials Science Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439 (Received 26 September 1997; revised manuscript received 9 March 1998)

An explanation is proposed of the unusual magnetoresistance, linear in magnetic field and positive, observed recently in nonstoichiometric silver chalcogenides. The idea is based on the assumption that these substances are basically gapless semiconductors with a linear energy spectrum. Most of the excess silver atoms form metallic clusters which are doping the remaining material to a very small carrier concentration, so that even in a magnetic field as low as 10 Oe, only one Landau band participates in the conductivity.



#### Symmetry operation of WTe<sub>2</sub>

- 1. C<sub>2</sub> (glide)
- 2. ky=0 mirror
- 3. kx=0 mirror (glide)
- 4. time-reversal

#### no inversion symmetry

### Weyl candidate

- No crystal inversion symmetry
- SOC band inversion gap on mirror plane











WPs distance = topological strength





B"





ARTICLE

Received 23 Sep 2015 | Accepted 7 Jan 2016 | Published 15 Feb 2016

DOI: 10.1038/ncomms10639

OPEN

Prediction of an arc-tunable Weyl Fermion metallic state in  $Mo_xW_{1-x}Te_2$ 

Tay-Rong Chang et al. Nat. Commun. 7, 10639 (2016)



topological strength can be tuned by varying Mo doping concentration

arXiv.org > cond-mat > arXiv:1604.07079

Condensed Matter > Mesoscale and Nanoscale Physics

Measuring Chern numbers above the Fermi level in the Type II Weyl semimetal Mo $_x$ W $_{1-x}$ Te $_2$ 

I. Belopolski ... T.-R. Chang et al



MoWTe<sub>2</sub> is a Weyl semimetal arXiv:1604.01706, arXiv:1603.08508, arXiv:1604.00139, arXiv:1604.04218, arXiv:1604.02116, and arXiv:1604.07079

### **Type-II Weyl semimetal**

# LETTER

doi:10.1038/nature15768

### **Type-II Weyl semimetals**

Alexey A. Soluyanov<sup>1</sup>, Dominik Gresch<sup>1</sup>, Zhijun Wang<sup>2</sup>, QuanSheng Wu<sup>1</sup>, Matthias Troyer<sup>1</sup>, Xi Dai<sup>3</sup> & B. Andrei Bernevig<sup>2</sup>



### Weyl+symmetry: High chiral charge



### Weyl+symmetry: High chiral charge



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## **Topological phases**



### Nodal-line semimetal: topology





### Strategy

(1) Layer structure avoid complex 3D band structure

### (2) Breaking either time-reversal symmetry or inversion symmetry

coexistence of both TR and I symmetries is too restrictive for a line touching to occur

#### (3) planer-like crystalline symmetry

rotational symmetry protect a part of band touching points

### Simplest case:

the crossing points of two paraboloids bands.



### **Nodal-line candidate**

**RAPID COMMUNICATIONS** 

#### PHYSICAL REVIEW B 89, 020505(R) (2014)

#### Noncentrosymmetric superconductor with a bulk three-dimensional Dirac cone gapped by strong spin-orbit coupling

Mazhar N. Ali,<sup>1,\*</sup> Quinn D. Gibson,<sup>1</sup> T. Klimczuk,<sup>2,3</sup> and R. J. Cava<sup>1,†</sup>

<sup>1</sup>Department of Chemistry, Princeton University, Princeton New Jersey, 08544, USA <sup>2</sup>Faculty of Applied Physics and Mathematics, Gdansk University of Technology, Narutowicza 11/12, 80-233 Gdansk, Poland <sup>3</sup>Institute of Physics, Pomeranian University, Arciszewskiego, 76-200 Slupsk, Poland (Received 30 October 2013; revised manuscript received 27 December 2013; published 14 January 2014)





## **Nodal-line candidate: PbTaSe<sub>2</sub>**



notal PbTaSe<sub>2</sub>

G. Bian, Tay-Rong Chang\* et al. Nat. Commun. 7, 10556 (2016)



## **Nodal-line (spinless): PbTaSe**<sub>2</sub>



### **NL evolve under SOC**



### **NL evolve under SOC**



### **NL evolve under SOC**



## **Nodal-line (spinful): PbTaSe**<sub>2</sub>



1: spin up along z axis 🗜 spin down along z axis •  $Pb-p_x/p_y$ •  $Ta-d_{xy}/d_{x2-y2}$ +,-: mirror eigenvalues

**Nodal lines are protected** by mirror symmetry!

Different eig. values = crossing

# **Nodal-line:** PbTaSe<sub>2</sub>



### **Nodal-line:** PbTaSe<sub>2</sub>



# Nodal-line PbTaSe<sub>2</sub>: topology

![](_page_55_Figure_1.jpeg)

# Nodal-line PbTaSe<sub>2</sub>: surface states

![](_page_56_Figure_1.jpeg)

![](_page_56_Picture_2.jpeg)

winding number

$$g/\rho = \pm 1$$

bulk-boundary correspondece

![](_page_56_Picture_6.jpeg)

![](_page_56_Figure_7.jpeg)

![](_page_56_Figure_8.jpeg)

![](_page_56_Figure_9.jpeg)

![](_page_56_Figure_10.jpeg)

-0.3 0.0 0.3

![](_page_56_Picture_12.jpeg)

# **Nodal-line PbTaSe<sub>2</sub>: Experiments**

![](_page_57_Figure_1.jpeg)

S.-Y. Guan, P.-J. Chen, M.-W. Chu et al, unpublished

#### STEM

![](_page_57_Picture_4.jpeg)

### **Nodal-line PbTaSe<sub>2</sub>: Experiments**

![](_page_58_Figure_1.jpeg)

### **Nodal-line PbTaSe<sub>2</sub>: Experiments**

![](_page_59_Figure_1.jpeg)

![](_page_59_Figure_2.jpeg)

![](_page_59_Figure_3.jpeg)

### **Nodal-line PbTaSe**<sub>2</sub>

![](_page_60_Figure_1.jpeg)

Ta

Pb

-0.8

1.5

### 

Editor: Nodal-line shape band appearing near Fermi level hosts unique properties in topological matter, which has yet to be confirmed in real materials. Here, the authors report the existence of topological nodal-line states in the non-centrosymmetric singlecrystalline spin-orbit semimetal PbTaSe<sub>2</sub>.

## **Nodal-line semimetal: TITaSe<sub>2</sub>**

![](_page_61_Figure_1.jpeg)